

## Review Article

# Effects of Cross Diffusion and Radiation on Magneto Mixed Convective Stagnation Flow from a Vertical Surface in Porous Media with Gyrotactic Microorganisms

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## Abstract

In this paper the Soret and Dufour effects on steady mixed convective boundary layer flow on a vertical surface embedded in porous medium subjected to a magnetic field containing gyrotactic microorganism is studied. The governing momentum, energy, concentration and microorganism equations are transformed into a set of coupled differential equations. The obtained equations are solved by MAPLE 14.0 algorithm and the numerical result for different values of Soret number  $S_r$ , Dufour number  $D_r$ , Lewis number  $Le$ , bioconvection Lewis number  $L_b$ , bioconvection Peclet number  $P_b$ , Hartmann number  $Ha^2$ , thermal radiation parameter  $R_\theta$  and buoyancy numbers is presented graphically for both assisting and opposing flow. Comparisons are made with available published results as special cases to validate numerical data and excellent compatibility is found. The effects of physical parameters on Nusselt number, Sherwood number and density of motile microorganism are also presented. It is observed that diffusion thermo and thermal diffusion effects on temperature, concentration and microorganisms profile distributions are quite opposite.

**Keywords:** Boundary Layer Flow; Double Diffusive; Slicing Analysis; Nonlinear ODE; Flow Solutions; Physical Parameters.

**Subject Classification:** 76Dxx

## Introduction

The mixed convection (combined free and forced) boundary layer flow has attracted considerable attention in recent years because of their wide applications in nuclear reactor technology transpiration cooling, separation process in aquifer, ground water pollution, oil recovery processes, food processing, drawing of plastic films, hot rolling and continuous casting of metals and spinning of fibers, glass-fiber and paper productions. Sakiadis [1,2] studied the boundary layer flow through a moving surface. Grubka and Bobba [3], Elbashbeshy [4] observed temperature field in the boundary layer flow over a stretching surface. Cheng [5] studied the mixed convection flow on inclined surfaces in a porous medium. The mixed convection along a nonisothermal wedge in a porous medium was studied by Kumari and Gorla [6]. The Neild and Bejan [7], Ingham and Pop [8,9], Vafai [10] have reported the detailed review of the convection through porous media. Now a days the large number of publications, Lai [11], Postelnicu [12], Chamka and Khaled [13], Bansod and Ambedkar [14], Srinivasachary and Surender [15], Garg [16], Swamy [17], of the mixed convective heat and mass transfer in porous media have been studied for vertical surfaces.

The study of microorganism is called microbiology. Great attentions are paid to biological fluid mechanism particular in the field of microbiology where developments of microorganisms (e.g. motile species of bacteria and algae) in bioconvection are studied. It is commonly observed that in fluid mechanics solid particles are either

carried by the fluid flow or pushed by external forces. Bio-convection patterns are inspected in cultures of swimming micro-organisms which are heavier than water and tend to propel themselves toward the upper surface of their environment in response to external stimuli such as gravity, light, and chemical gradient. Bioconvection can also be classified as the macroscopic fluid motion because of the density gradient associated with swimming micro-organisms [18–22] which intensify the density of base fluid in a specific direction that causes bioconvection flow.

Scientists formulate models to observe the fluid motions associated with aquatic microorganisms. A large number of mechanisms are developed that force microorganisms to swim in specific directions depending on the ambient condition. There are different types of motile micro-organisms such as oxytactic or chemotaxis, negative gravitaxis and gyrotactic micro-organisms according to their impelling behavior. Neild and Bejan [7] defined bioconvection as “Pattern formation in suspensions of microorganisms such as bacteria and algae due to up swimming of the microorganisms”. Gyrotactic microorganisms such as *Cnivalis* swim upward in still water due to the fact that their center of mass is located behind their center of buoyancy. The behavior of mixed convection in suspensions of gyrotactic/oxytactic microorganisms is discussed widely in [23–27].

Nowadays analyzing Soret and Dufour effects on convective processes have sprung researchers’ interests. Thermal diffusion or Soret effect is a mass flux due to temperature gradient [28]. The

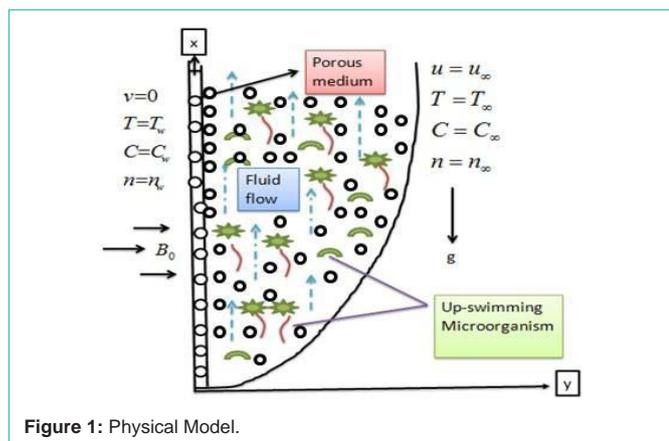


Figure 1: Physical Model.

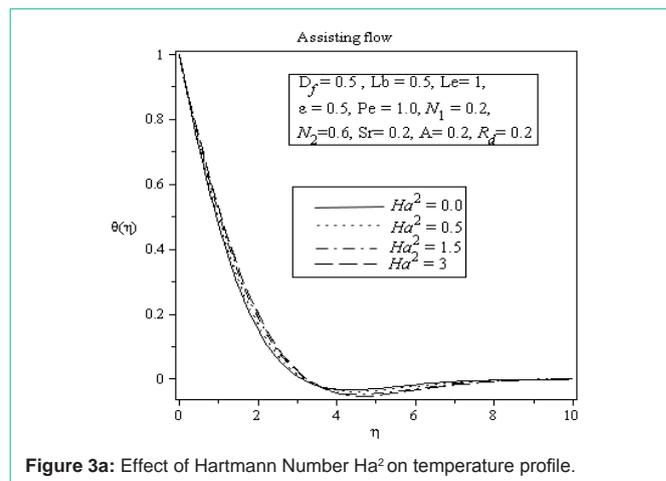


Figure 3a: Effect of Hartmann Number  $Ha^2$  on temperature profile.

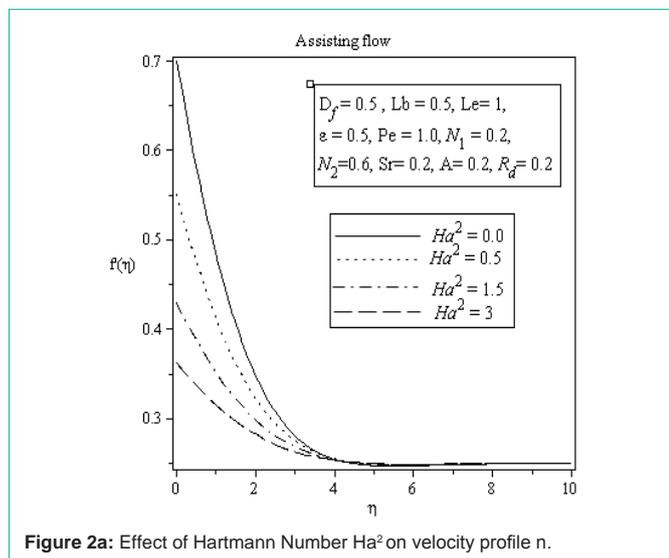


Figure 2a: Effect of Hartmann Number  $Ha^2$  on velocity profile  $n$ .

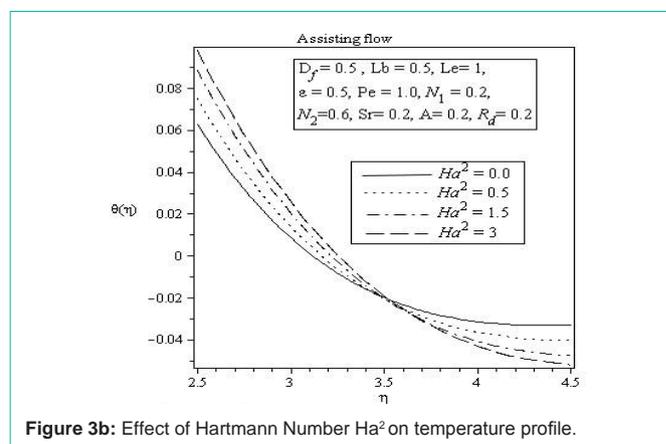


Figure 3b: Effect of Hartmann Number  $Ha^2$  on temperature profile.

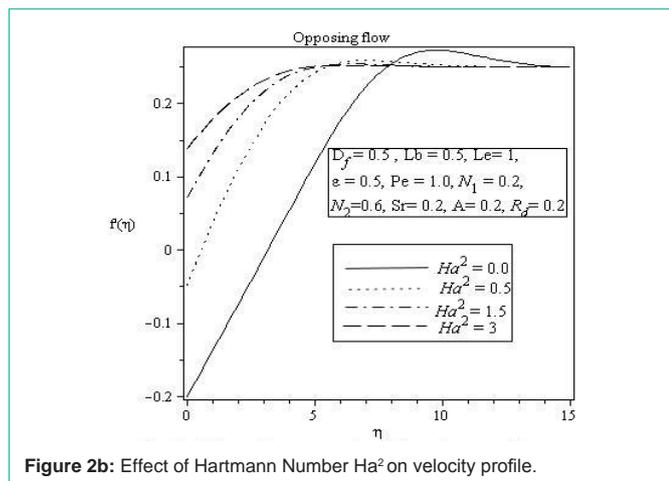


Figure 2b: Effect of Hartmann Number  $Ha^2$  on velocity profile.

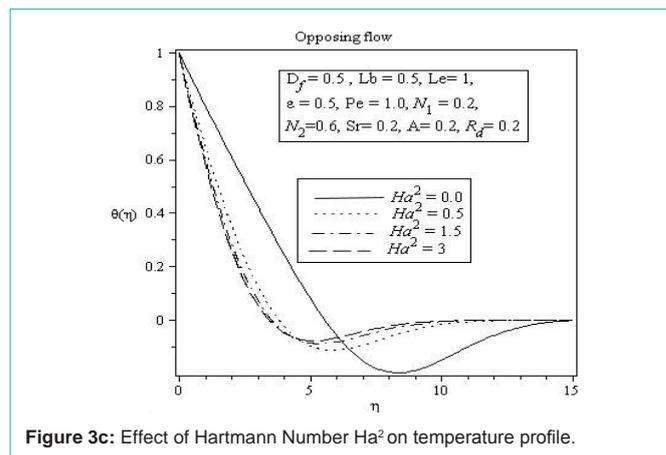


Figure 3c: Effect of Hartmann Number  $Ha^2$  on temperature profile.

higher the temperature gradient, the larger the Soret effect is. Dufour effect is enthalpy flux due to concentration gradient. As observed in a moving fluid, energy flux can be generated by both temperature and concentration gradient which causes the Dufour (thermo-diffusion) and the Soret (thermal-diffusion) effects. Since in most cases, Dufour and Soret effects are in smaller order of magnitudes than thermo-fluid

effects described by Fourier's and Fick's laws, they are neglected in many studies. However, Eckert and Drake [29], Mortimer and Eyring [30] describe several cases where diffusion-thermo and thermal-diffusion effects are quite significant. Cheng [31], Srinivasacharya [32] presented Soret and Dufour effects on mixed convection heat and mass transfer in porous medium. El -Aziz [33] studied effects of Soret and Dufour parameters on velocity, concentration and temperature profiles. Mohammed Ahmed [34] showed the effects of cross-diffusion and radiation on mixed convection from a vertical plate in a fluid saturated porous medium.

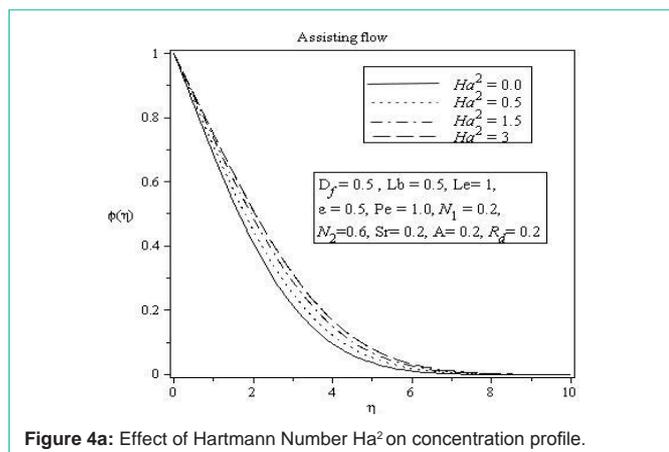


Figure 4a: Effect of Hartmann Number  $Ha^2$  on concentration profile.

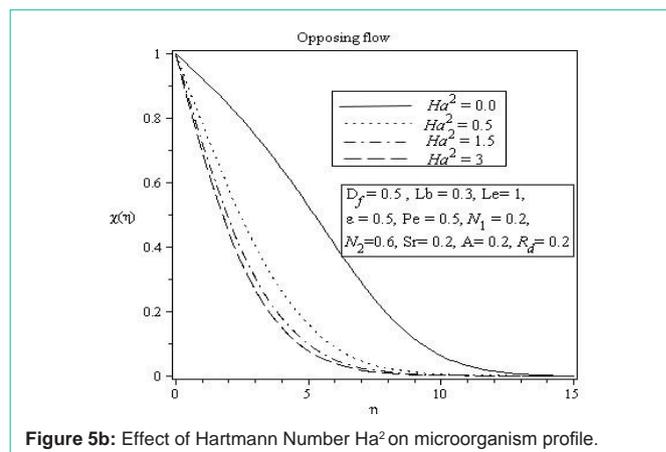


Figure 5b: Effect of Hartmann Number  $Ha^2$  on microorganism profile.

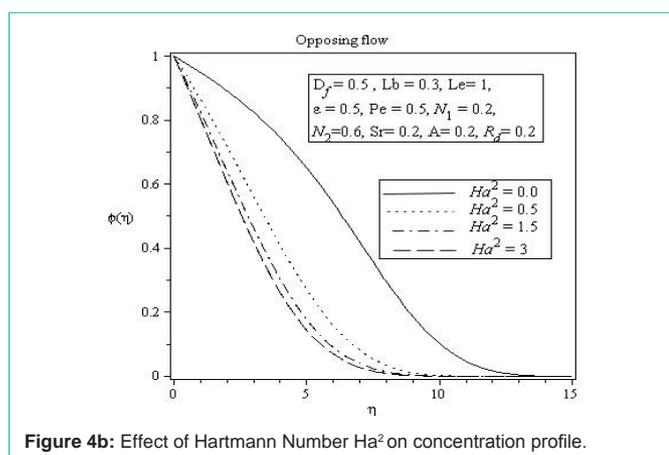


Figure 4b: Effect of Hartmann Number  $Ha^2$  on concentration profile.

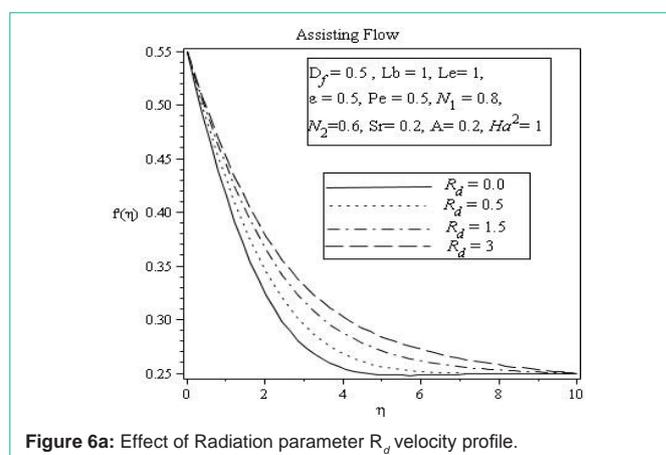


Figure 6a: Effect of Radiation parameter  $R_d$  velocity profile.

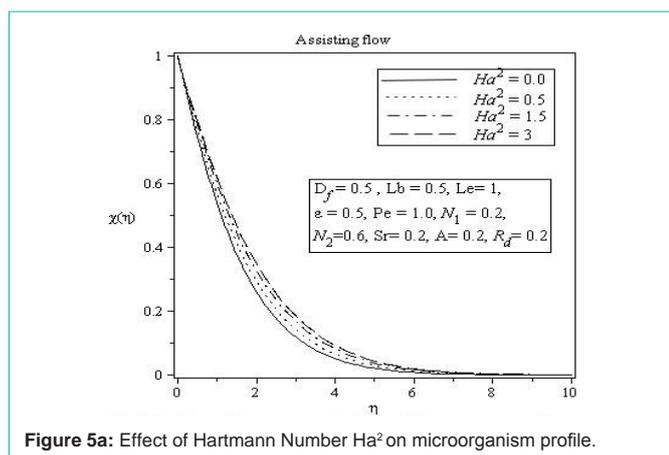


Figure 5a: Effect of Hartmann Number  $Ha^2$  on microorganism profile.

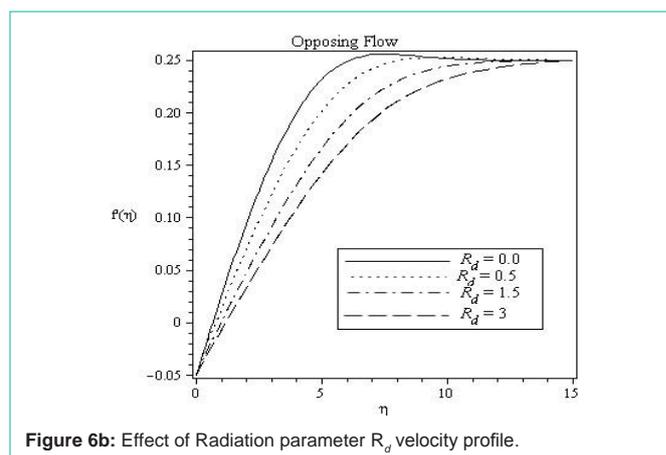


Figure 6b: Effect of Radiation parameter  $R_d$  velocity profile.

The study of magnetic field has been a focused in recent researches due to its wide applications in many engineering fields such as in petroleum engineering, chemical engineering, composite or ceramic engineering, biochemical engineering and heat exchangers. The applications of magnetic field to convection processes also constitute a major field of interest (Magneto-Hydrodynamic or MHD) in many areas of engineering and physical sciences such as crystal growth, metal casting and liquid metal cooling blankets for fusion reactors. The effect of a magnetic field on convective fluid flow in porous media has been investigated in the following studies, Postelnicu [35]

investigated on influence of magnetic field on natural convection considering Soret and Dufour effects. Zhang e [36] investigated MHD radiative flow of nanofluid over a surface with variable surface heat flux and chemical reaction. Khan [37] analyzed non-aligned MHD radiative flow of variable viscosity nanofluid past a stretching surface. Effect of inclined magnetic field in third grade fluid flow with variable thermal conductivity is presented by Hayat [38]. Alsaedi [39] studied MHD bioconvective flow in the presence of nanofluid containing microorganism.

Our aim in this paper is to analyze a steady mixed convective

flow containing gyrotactic microorganisms over a vertical plate in porous medium coexistence diffusion thermo, thermal diffusion effects in the presence of external magnetic field. The behavior of entire boundary layer thickness of velocity, temperature, concentration and microorganism profile is observed for different values of bioconvection parameters. The effect of Nusselt number, Sherwood number as well as density of motile microorganism on increasing value of bioconvection parameters, Soret and Dufour number, Hartmann number, radiation parameter are investigated in this study. The impact on different profile distributions are analyzed separately for natural, forced and also mixed convection regimes.

### Mathematical Formulation

Consider the problem of two-dimensional steady-state boundary layer flow of hydromagnetic fluid containing gyrotactic microorganisms. Due to gravity the fluid and microorganisms both fall downwards along a vertical flat plate. The coordinate system was chosen such that x-axis is along the plate and y-axis is normal to it. The schematic sketch of the problem is shown in Figure 1. The vertical wall is kept at constant distributions such as temperature  $T_w$ , the fluid volume fraction  $C_w$  and the density of motile microorganisms  $n_w$ , which are considered to be greater than the ambient temperature and concentrations  $T_\infty$ ,  $C_\infty$  and  $n_\infty$ .

The governing partial differential equations related to conservation of mass, momentum, energy, oxygen and microorganisms using the Oberbeck-Boussinesq approximation can be written as,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$(1 + \frac{\sigma\beta_0^2 k}{\rho\nu}) \frac{\partial u}{\partial y} = \pm \frac{gk}{\nu} (\beta_r \frac{\partial T}{\partial y} + \beta_c \frac{\partial C}{\partial y} + \beta_n \frac{\partial n}{\partial y}) \tag{1}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{D_m k_T}{C_s C_p} \frac{\partial^2 C}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y} \tag{2}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} + \frac{D_m k_T}{T_m} \frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$u \frac{\partial n}{\partial x} + v \frac{\partial n}{\partial y} + \frac{bW_c}{(C_w - C_\infty)} \left[ \frac{\partial}{\partial y} \left( n \frac{\partial C}{\partial y} \right) \right] = D_n \frac{\partial^2 n}{\partial y^2} \tag{4}$$

With the boundary conditions are of the form:

$$v = 0, T = T_w, C = C_\infty, n = n_\infty \text{ at } y = 0 \tag{5}$$

$$u \rightarrow u_\infty, T \rightarrow T_\infty, C \rightarrow C_\infty, n \rightarrow n_\infty \text{ at } y \rightarrow \infty \tag{6}$$

In equation (2), plus (+) sign indicates assisting flow and minus (-) sign indicates opposing flow. Using Rosseland approximation (Raptis [40] and Sparrow [41]) radiative heat flux term can be defined as,

$$q_r = -\frac{4\sigma^*}{3k} \frac{\partial T^4}{\partial y} \tag{7}$$

where  $\sigma^*$  and  $k$  are the Stefan-Boltzman constant and the mean absorption co-efficient.

Introducing following dimensionless quantities

$$n = \frac{y}{x} Ra_x^{\frac{1}{2}} \left( 1 + \frac{Pe_x^{\frac{1}{2}}}{Ra_x^{\frac{1}{2}}} \right), \psi = \alpha Ra_x^{\frac{1}{2}} \left( 1 + \frac{Pe_x^{\frac{1}{2}}}{Ra_x^{\frac{1}{2}}} \right) f(\eta)$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \chi(\eta) = \frac{n - n_\infty}{n_w - n_\infty} \tag{8}$$

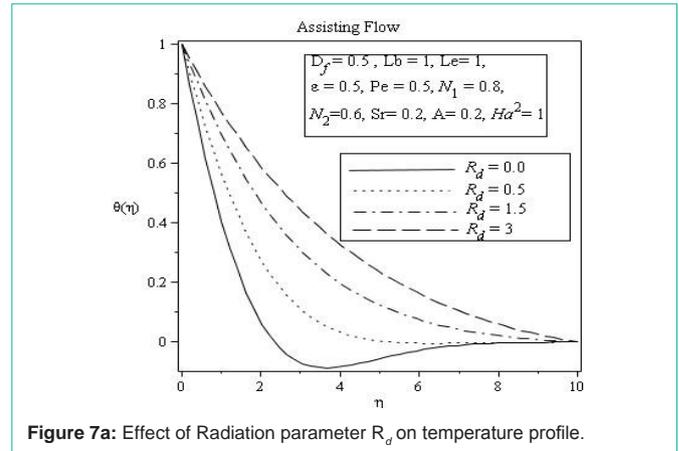


Figure 7a: Effect of Radiation parameter  $R_d$  on temperature profile.

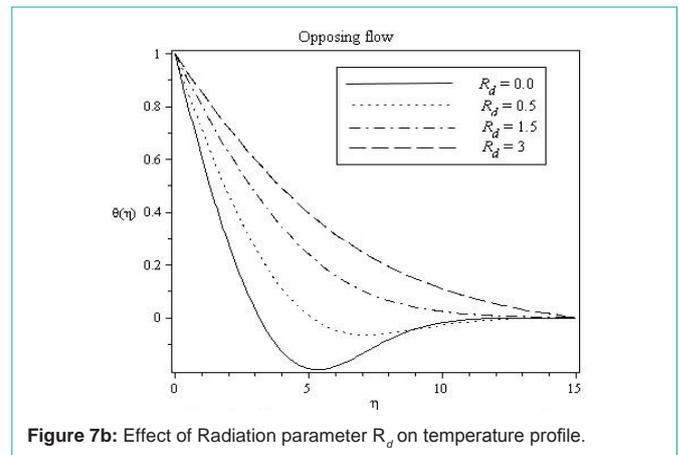


Figure 7b: Effect of Radiation parameter  $R_d$  on temperature profile.

By defining stream function as follow

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \tag{9}$$

The governing equations are

$$(1 + Ha^2) f'' = \pm (1 - \epsilon) [\theta' + N_1 \phi' + N_2 \chi'] \tag{10}$$

$$\left( 1 + \frac{4}{3} R_d \right) \theta'' + \frac{1}{2} f \theta' + D_f \phi'' = 0 \tag{11}$$

$$\phi'' + \frac{Le}{2} f \phi'' = 0 \tag{12}$$

$$\chi'' + \frac{Lb}{2} f \phi' - Pe[\phi \chi' + (\chi + A)\phi''] = 0 \tag{13}$$

The transformed boundary conditions are

$$\eta = 0, f = 0, \theta = 1, \chi = 1 \text{ and } \tag{14}$$

$$\eta \rightarrow \infty, f' \rightarrow \epsilon^2, \theta \rightarrow 0, \chi \rightarrow 0 \tag{15}$$

Where,

$$N_1 = \frac{\beta_c (C_w - C_\infty)}{\beta_r (T_w - T_\infty)}, N_2 = \frac{\beta_n (n_w - n_\infty)}{\beta_r (T_w - T_\infty)}, Ha^2 = \frac{\sigma B_0^2 k}{\rho \nu}, Ra_x = \frac{kg \beta_r (T_w - T_\infty) x}{\nu \alpha}$$

$$Pe_x = \frac{u_\infty x}{\alpha}, A = \frac{n_\infty}{n_w - n_\infty}, R_d = \frac{4\sigma^* T_\infty^3}{kk^*}, D_f = \frac{D_m k_T}{\alpha}, \epsilon = \frac{1}{1 + \frac{Ra_x^{\frac{1}{2}}}{Pe_x^{\frac{1}{2}}}} \tag{16}$$

It's noteworthy that  $\epsilon=0$  ( $Pe_x=0$ ) and  $\epsilon=1$  ( $Ra_x=0$ ) respectively

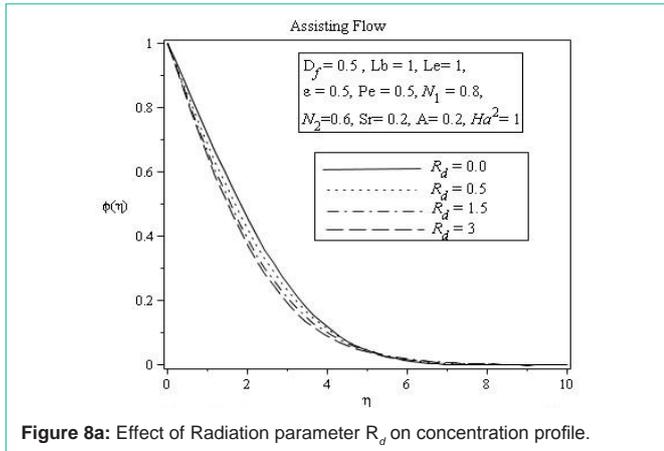


Figure 8a: Effect of Radiation parameter  $R_d$  on concentration profile.

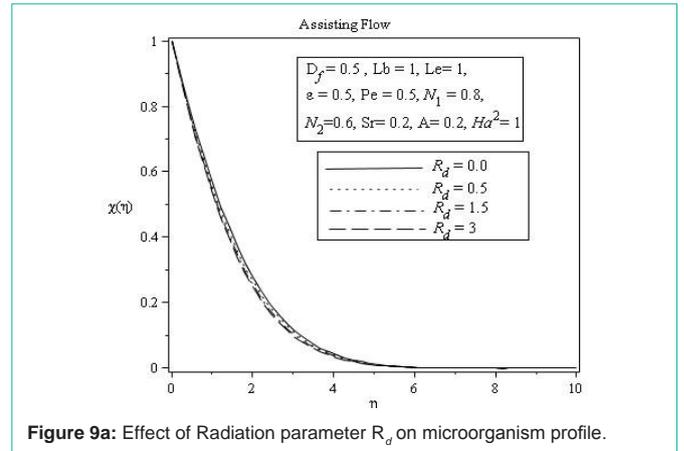


Figure 9a: Effect of Radiation parameter  $R_d$  on microorganism profile.

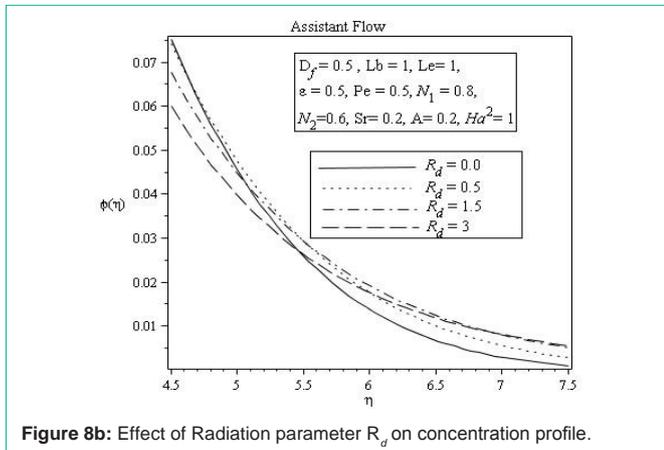


Figure 8b: Effect of Radiation parameter  $R_d$  on concentration profile.

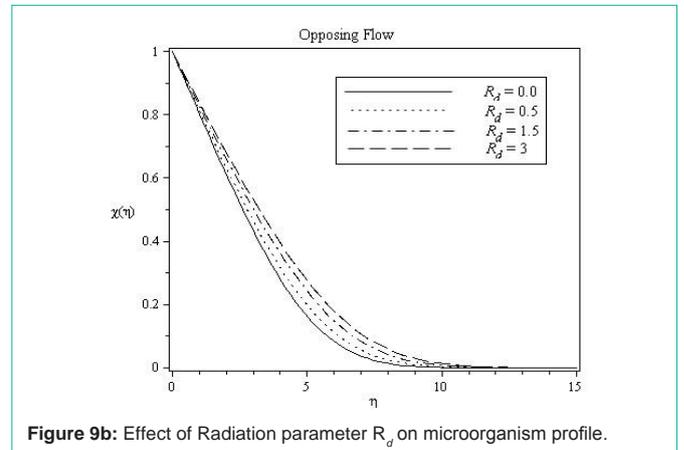


Figure 9b: Effect of Radiation parameter  $R_d$  on microorganism profile.

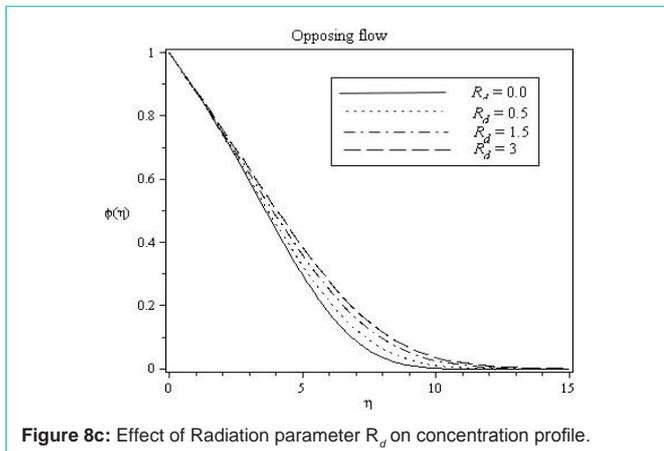


Figure 8c: Effect of Radiation parameter  $R_d$  on concentration profile.

corresponds to pure free convection, pure forced convection. So any value between  $\epsilon=0$  to  $\epsilon=1$  corresponds to mixed convection regime.

The quantities of practical interest in this study are the local Nusselt number  $Nu_x$ , the Sherwood number  $Sh_x$  and the local density number of the motile microorganisms  $Nn_x$  are defined as

$$Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, Sh_x = \frac{xq_m}{D(C_w - C_\infty)}, Nn_x = \frac{xq_n}{D_n(n_w - n_\infty)} \quad (18)$$

Where  $q_w$ ,  $q_m$ , and  $q_n$  are the wall heat, the wall mass and wall motile microorganisms fluxes, respectively and are defined as

$$q_w = -k \left( \frac{\partial T}{\partial y} \right) y = 0, q_m = -D_B \left( \frac{\partial C}{\partial y} \right) y = 0, q_n = -D_n \left( \frac{\partial n}{\partial y} \right) y = 0 \quad (19)$$

Using variables (9),(17),(18) and (19), we obtain

$$\epsilon Pe_x^{-1} Nu_x = -\theta'(0), \epsilon Pe_x^{-1} Sh_x = -\psi'(0), \epsilon Pe_x^{-1} Nn_x = -\chi'(0) \quad (20)$$

where  $\epsilon Pe_x^{-1} Nu_x, \epsilon Pe_x^{-1} Sh_x, \epsilon Pe_x^{-1} Nn_x$  are called the Nusselt number, Sherwood number local density number of the motile microorganisms respectively.

**Numerical solution**

The governing partial differential equations (1-7) were transformed into ordinary differential equations (11-16) respectively using similarity solutions and then numerically solved by using Maple 14.0 with the help of dsolve command. The type of BVP or IVP problems were identified and appropriate algorithms were implemented. The reliability and accuracy of Maple’s algorithm have been repeatedly verified in several recent research papers. For the further confirmation present results for the special cases are compared with some published results investigated by Hsieh [40], Ching-Yang Cheng[41], Gorla [42], A.J.Chamka [43].The asymptotic boundary conditions given by equation (15-16) were replaced by using a value of 15 for the similarity variable  $\eta_{max}$  as follows:  $\eta_{max}=15, f(15)=0,$

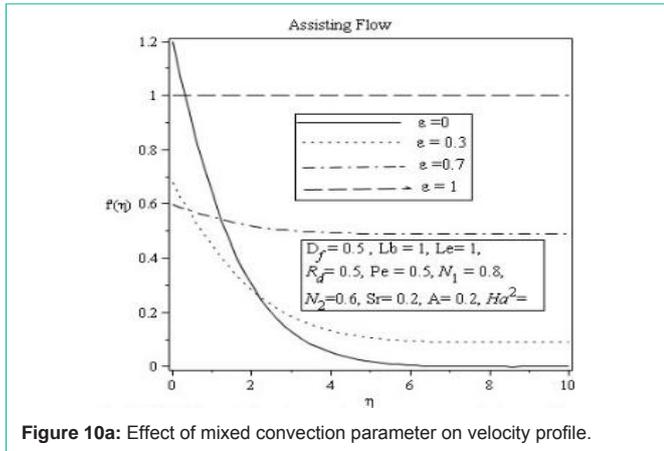


Figure 10a: Effect of mixed convection parameter on velocity profile.

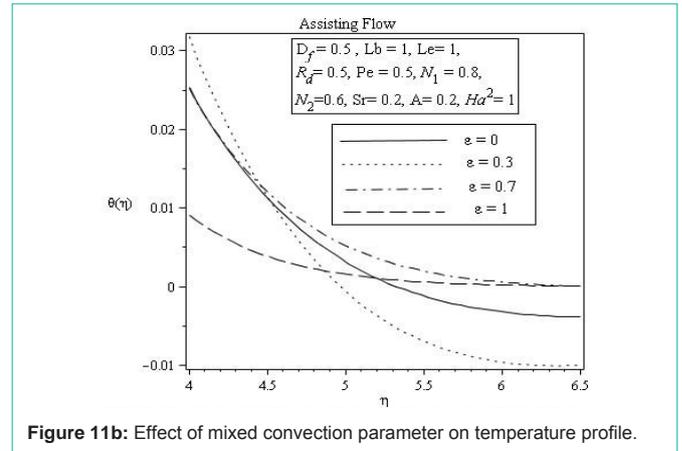


Figure 11b: Effect of mixed convection parameter on temperature profile.

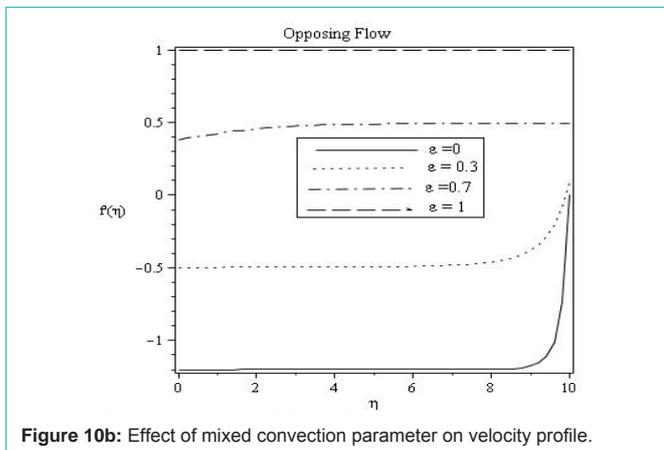


Figure 10b: Effect of mixed convection parameter on velocity profile.

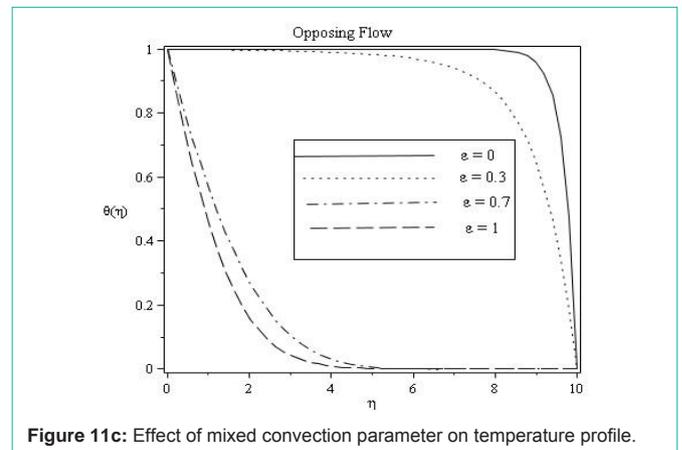


Figure 11c: Effect of mixed convection parameter on temperature profile.

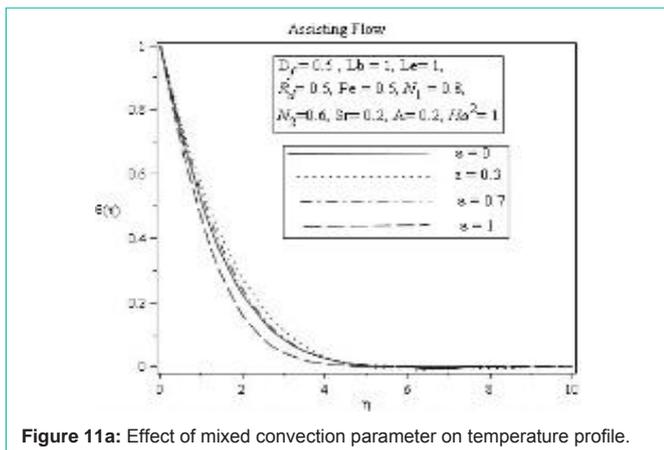


Figure 11a: Effect of mixed convection parameter on temperature profile.

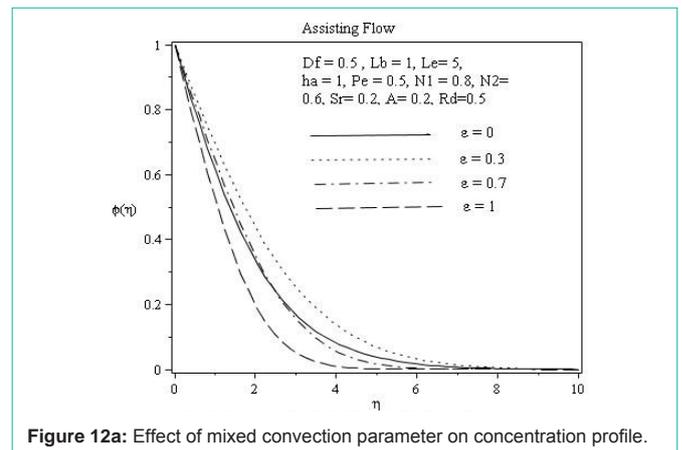


Figure 12a: Effect of mixed convection parameter on concentration profile.

$\theta(15)=0, \varphi(15)=0, \sigma(15)=0.$

### Result and Discussion

In figure (2-5) Effect of Hartmann number on velocity, temperature, concentration and microorganism profile is observed for  $D_f=0.5, L_b=0.5, L_e=1, \epsilon=0.5, Pe=1.0, N_1=0.2, N_2=0.6, Sr=0.2, A=0.2, R_d=0.2$ . The behavior of different profiles in all case is observed for assisting and also opposing flow. It is well known that the influence of magnetic field can be represented by Hartmann number. The Lorentz force rises if transverse magnetic field is placed normal to the flow

direction. This resistive  $-$ type force declines the motion of the fluid flow and increases the temperature of flow. As a result boundary layer thickness of concentration and microorganism profile increases with the increasing value of Hartmann number. In Figure 3(b) we can see thermal boundary layer thickness increases in the similar way according to increasing value of Hartmann number but after point  $\eta=3.513$  thermal boundary layer decreases. For all profiles opposite effects are observed for opposing Flow.

In Figure (6-9) effect of radiation parameter is observed  $D_f=0.5, L_b=1, L_e=1, \epsilon=0.5, Pe=0.5, N_1=0.8, N_2=0.6, Sr=0.2, A=0.2, Ha^2=1$ . In

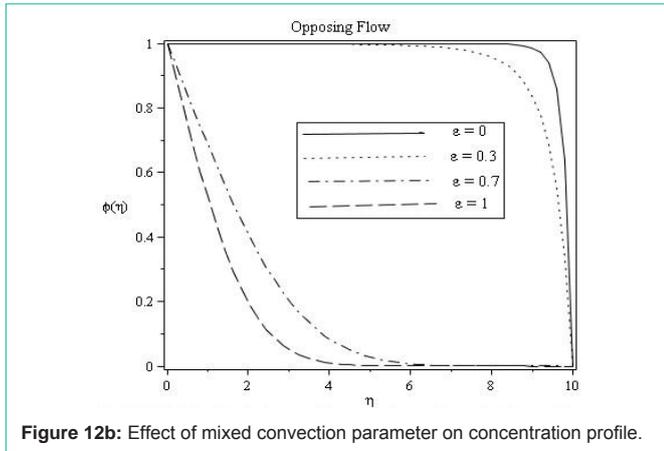


Figure 12b: Effect of mixed convection parameter on concentration profile.

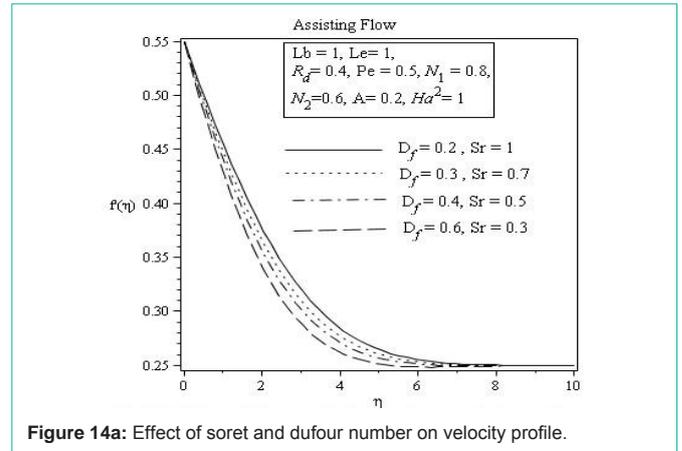


Figure 14a: Effect of solet and dufour number on velocity profile.

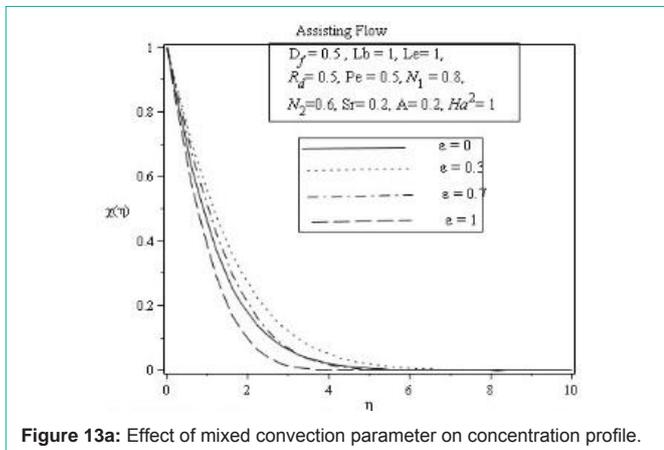


Figure 13a: Effect of mixed convection parameter on concentration profile.

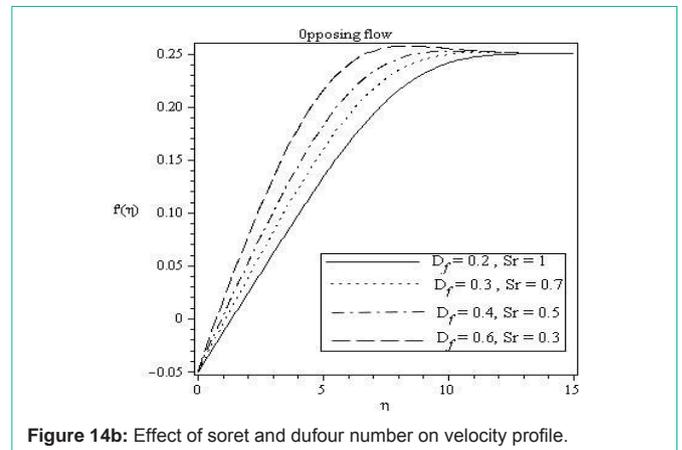


Figure 14b: Effect of solet and dufour number on velocity profile.

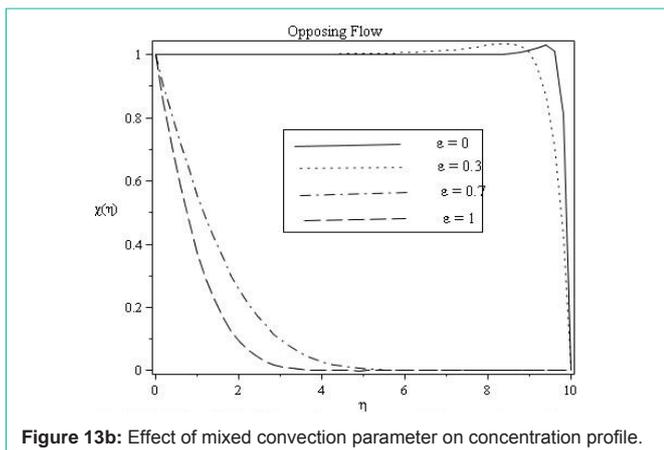


Figure 13b: Effect of mixed convection parameter on concentration profile.

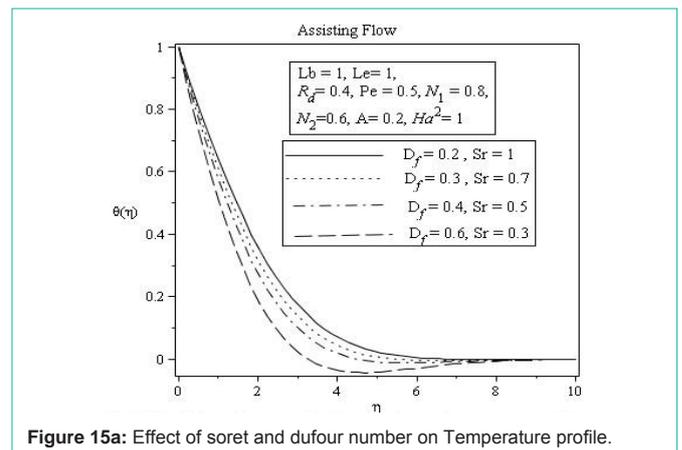


Figure 15a: Effect of solet and dufour number on Temperature profile.

6(a) the velocity profile and corresponding boundary layer thickness increases with the increasing of radiation parameter  $R_d$ . As higher values of radiation parameter implies higher surface heat flux, in Figure 7(a-b) it is observed that thermal boundary layer thickness greatly increases for assisting flow and also for opposing flow. When radiation parameter is negligible temperature profile decreases, but after  $\eta \approx 3.611$  it increases for assisting flow. The concentration and microorganism profile decreases with the increases of radiation parameter in Figure 8(a) to 9(b).

Effect of different values of mixed convection parameter is

observed for the values  $D_f=0.5, Lb=1, Le=1, R_d=0.5, Pe=0.5, N_1=0.8, N_2=0.6, Sr=0.2, A=0.2, Ha^2=1$  in figure (10-13). With the increasing of mixed convection parameter velocity profile increases. So momentum boundary layer thickness gradually increases from free convection to forced convection regime. For the influence of mixed convection parameter number of stagnation point flow observed for temperature profile 11(a). It can be seen clearly on 11(b). For  $\epsilon=0.7, \epsilon=1$  temperature, concentration and also microorganism profile remains almost similar to assisting flow in case of opposing flow but for  $\epsilon=0, \epsilon=0.3$  the characteristic of temperature, concentration

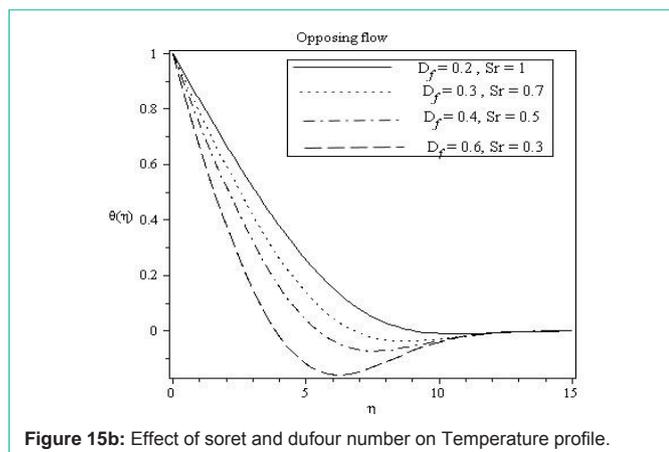


Figure 15b: Effect of solet and dufour number on Temperature profile.

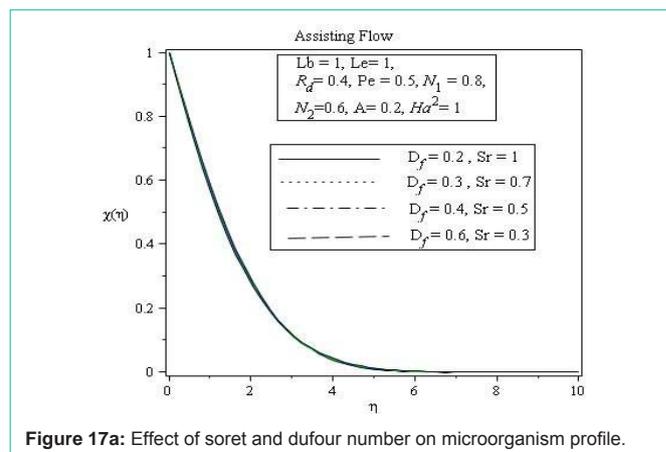


Figure 17a: Effect of solet and dufour number on microorganism profile.

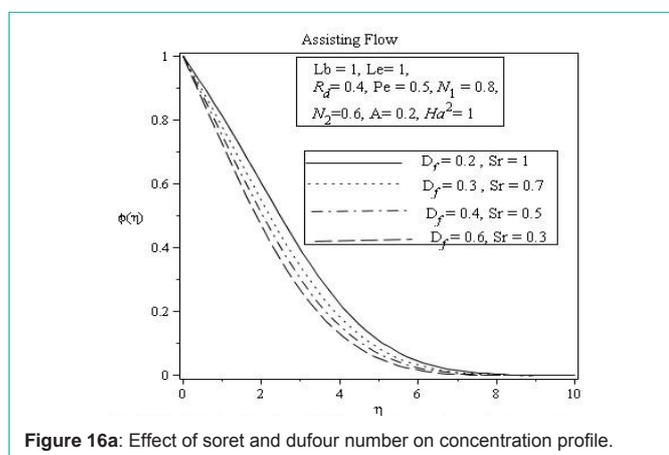


Figure 16a: Effect of solet and dufour number on concentration profile.

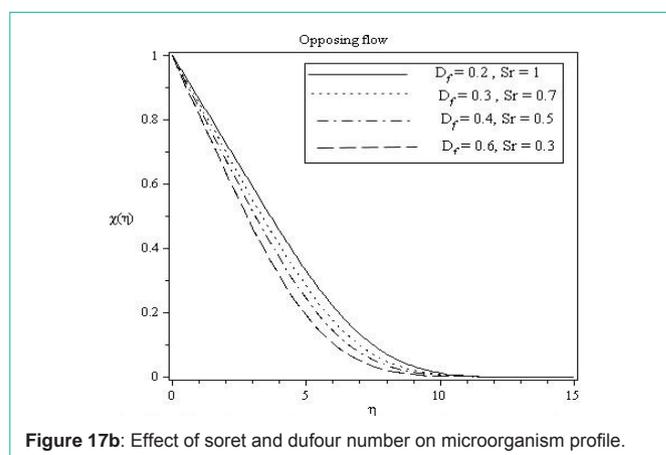


Figure 17b: Effect of solet and dufour number on microorganism profile.

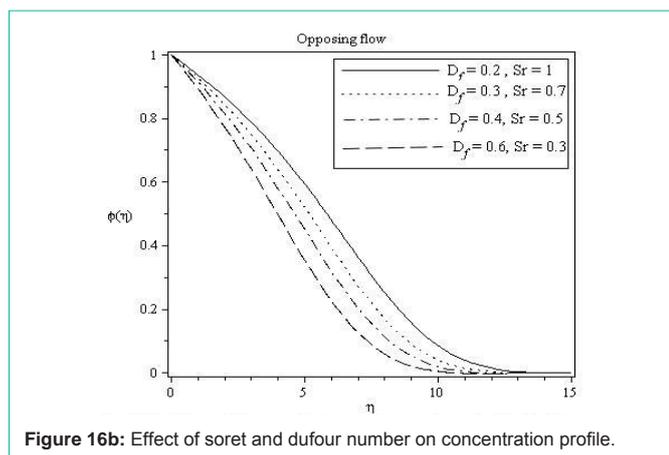


Figure 16b: Effect of solet and dufour number on concentration profile.

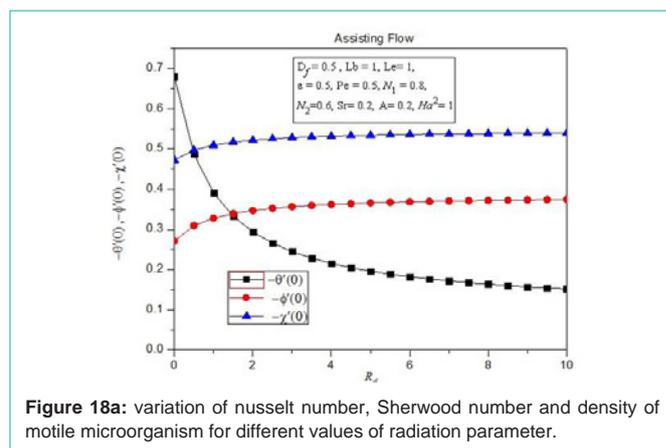


Figure 18a: variation of nusselt number, Sherwood number and density of motile microorganism for different values of radiation parameter.

and microorganism profile is quite opposite. In 12(a) and 13(a) it is observed that concentration and microorganism profile decreases with higher values of mixed convection parameter.

In figure 14(a) to 17(b) it is observed that all the profile distributions are decreasing with the increasing of Dufour number and decreasing of Soret number. So it can be stated that Soret and dufour effects on velocity, temperature, concentration and microorganism profile are quite opposite. Here another noticeable thing is except velocity profile figure 14(b) in all figure: 15(b), 16(b), 17(b) boundary layer thickness are decreasing for opposing flow similar to assisting flow.

Effect of Hartmann number  $Ha^2$  on nusselt number  $-\theta'(0)$ , Sherwood number  $(-\phi)'(0)$ , density of motile microorganism  $-\chi'(0)$  is observed in Table 1. With the increasing of Hartmann number nusselt number, Sherwood number and density of motile microorganism decrease for assisting flow and decrease for opposing flow. In Table 2 nusselt number  $-\theta'(0)$ , Sherwood number  $-\phi'(0)$ , density of motile microorganism  $-\chi'(0)$  increases for assisting flow with the increasing of dufour number and decreasing of Soret number. Opposite behavior is observed for opposing flow. Effect of radiation parameter is shown graphically in Figure 18(a) and 18(b). In 18(a) nusselt number

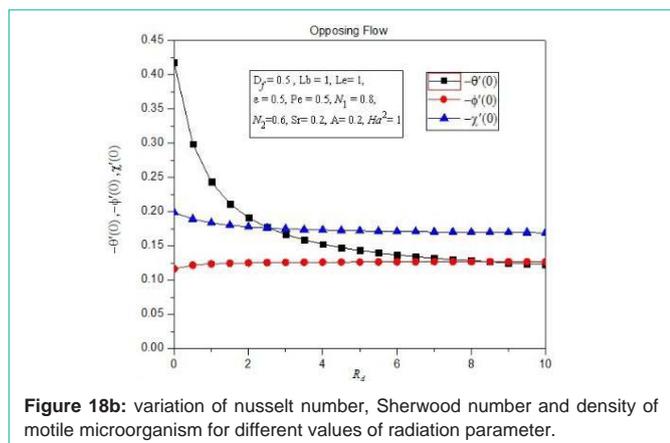


Figure 18b: variation of nusselt number, Sherwood number and density of motile microorganism for different values of radiation parameter.

Table 1: Effect of Hartmann number on Heat and mass transfer and density of motile microorganism for  $D_f=0.5, Le=1, Lb=1, \epsilon=0.5, Pe=0.5, A=0.2, N_1=0.8, N_2=0.6, R_d=0.5$  for assisting and opposing flow.

	Ha <sup>2</sup>	Assisting flow	Opposing flow
<b>-θ'(0)</b>	0	0.54162	0.01381
	0.5	0.50653	0.2054
	1	0.48715	0.29923
	1.5	0.47477	0.33193
	2	0.46615	0.34987
<b>-φ'(0)</b>	0	0.37407	0.0025
	0.5	0.33226	0.06189
	1	0.30962	0.12197
	1.5	0.29536	0.14883
	2	0.28552	0.16481
<b>-χ'(0)</b>	0	0.59965	0.00232
	0.5	0.53351	0.08045
	1	0.49752	0.18941
	1.5	0.47477	0.2354
	2	0.45906	0.2623

decreasing, Sherwood number and density of motile microorganism increasing. Similar effect is observed for nusselt number in 18(b) for opposing Flow. Some of our obtaining result has been compared with some published results which are presented in Table 3 and Table 4.

### Conclusion

In this study we have considered mixed convective flow over a vertical plate in porous media with Gyrotactic microorganisms to analyze the effect of cross diffusion, thermal radiation on velocity, Temperature, concentration and microorganism profile. The results obtained from our analysis are as follows:

For assisting flow with the increasing of  $Ha^2, D_f$  and decreasing of  $sr$  velocity profile decreases, on the other hand velocity profile increases with the increasing of  $R_d$  and mixed convection parameter  $\epsilon$ .

For both assisting and opposing flow stagnation flow is observed for temperature profile. It increases with the increasing of  $Ha^2, D_f$  and also  $\epsilon$ . In case of opposing flow for temperature profile remains

Table 2: Effect of Soret and Dufour number on Heat and mass transfer and density of motile microorganism for  $Ha^2=1.0, Le=1, Lb=1, \epsilon=0.5, Pe=0.5, A=0.2, N_1=0.8, N_2=0.6, R_d=0.4$  for assisting and opposing flow.

	$D_f$	$S_r$	Assisting flow	Opposing flow
<b>-θ'(0)</b>	0.1	1	0.34068	0.15161
	0.5	0.8	0.495	0.25781
	0.8	0.5	0.62259	0.37433
	1	0.1	0.7373	0.52759
<b>-φ'(0)</b>	0.1	1	0.20039	0.08757
	0.5	0.8	0.13637	0.04029
	0.8	0.5	0.16677	0.05345
	1	0.1	0.30958	0.14537
<b>-χ'(0)</b>	0.1	1	0.45492	0.15046
	0.5	0.8	0.41834	0.13948
	0.8	0.5	0.4259	0.16241
	1	0.1	0.48917	0.22396

Table 3: Comparison of heat transfer rate for the values  $Ha^2=0.0, Le=0, Lb=0, Pe=0.0, A=0.0, N_1=0.0, N_2=0.0, R_d=0.0$  for pure free convection  $\epsilon=0.0$ .

	Hsieh et al (42) when $\epsilon=0$	Ching-Yang Cheng-43 when $\epsilon=0$	Present
<b>-θ'(0)</b>	0.4438	0.4438	0.4439

Table 4: Comparison of  $f'(0)$  and  $-\theta'(0)$  for the values  $Ha^2=0.0, Le=0, Lb=0, Pe=0.0, A=0.0, N_1=0.0, N_2=0.0, R_d=0.0$  for pure forced convection  $\epsilon=1.0$ .

	Gorla et al (44) when $B=0$	A.J.Chamka(45) When $B=0$	Present
<b>f'(0)</b>	1	1	1
<b>-θ'(0)</b>	0.58398	0.5644	0.5641

unchanged in forced convection regime. For  $R_d$  temperature profile decreases.

Concentration profile increases with  $Ha^2$  and  $s_r$ , decreases with the increasing values of  $D_f$ . Stagnation flow is observed for  $R_d$  and  $\epsilon$ . For opposing flow concentration profile has no effect in forced convection regime.

For assisting flow microorganism profile increases with  $Ha^2$  and  $R_d$ . Soret and Dufour number have very negligible effect on microorganism profile.

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