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# **Review Article**

# The Organization of Multi-Flow Heat Exchange Systems with Minimal Irreversibility

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# Annotation

The study estimates minimal entropy production, the corresponding distribution of heat exchange surfaces and contact temperatures for heat exchange systems with fixed total heat load and constant heat transfer coefficients for all system. Mathematical conditions to reach this minimal entropy production are found. For a typical heat exchange system entropy production can be expressed through flow's temperatures. Comparison of the actual entropy production with its minimal value leads to estimates of thermodynamic feasibility and efficiency of the system.

## **List of Symbols**

C: Specific Heat (J/K kg); g: Flow rate (mole/s); j,i: Index; S: Entropy (J/K); T,u: Temperature (K); q: Heat load (J/s); W: Water equivalent (J/K s)

Greek symbols:  $\alpha$ : Heat transfer coefficient (J/K s);  $\sigma$ : Entropy production (J/K s);  $\lambda$ : Lagrange coefficient

# Introduction

Classical estimates for limiting performance of technological systems (heat and refrigerating machines, separation systems, chemical reactors, etc.) are based on efficiency limits of reversible processes (Carnot efficiency, reversible work of separation, standard chemical affinity, etc.). These estimates are very important but cannot be reached by real systems as the reversible models do not take into account effects of flow intensity, contact surfaces and some other factors related to a fixed productivity and finite dimensions of the equipment. In some cases reversible estimations lose any sense, as in the case of stationary non-equilibrium systems with several reservoirs or systems with substance and energy in flows. The heat exchanger is an example of such a system. Therefore, thermodynamic estimations of heat exchanger efficiency must take into account the effect of a restricted contact surface (integral coefficient of heat exchange) and of the finite heat load (that is, the finite amount of heat transmitted from hot to cold streams per unit of time). For the efficiency estimation of such systems we use the exergy method (see [1,2] etc.), that is the method of rating the system's exergy loss. This exergy loss is proportional to the entropy production and to the ambient temperature T<sub>0</sub>. The minimum exergy loss at given temperatures of hot streams and for a given heat load corresponds to the maximum average temperatures of cold streams at the outlet of the heat exchanger.

In this study we have solved the problem of the minimal entropy production (minimal exergy losses) in some typical multi- flow heat exchange systems. Such estimation permits to:

• Define the significance of such factors as temperatures and heat capacities streams, heat loads, an integral coefficients of the heat exchange, etc., in a selected heat exchange system;

• Exploit the optimal heat exchange conditions to make a designed system's configuration closer to the ideal when designing new systems;

• Evaluate thermodynamic efficiencies of the designed system by means of comparison of its actual entropy production with the least possible entropy production under the same conditions.

The entropy production in a heat exchange system can be calculated for known fluxes of heat capacity rate (water equivalent), input and output temperatures of these streams. Stream's water equivalent is product of flow rate g [kg/s] and specific heat C [J/kg K] (W = gC). Alternatively, one can also calculate the sum of products of heat loads q and corresponding driving forces, say  $\overline{X}$ , for each heat exchanger (see below). By assumption each driving force is a monotonously increasing function of  $\overline{q}$  (usually it is proportional to  $\overline{q}^2$ ). In the latter case the entropy production is proportional to  $\overline{q}^2$  and inversely proportional to the product of heat coefficient  $\overline{a}$  and contact area, S. For a constant system's exchange area, S, this entropy production could be arbitrarily small if the heat load  $\overline{q}$  is arbitrarily small and the heat coefficient  $\overline{a}$  is fixed or if the coefficient  $\overline{a}$  is arbitrarily large and the load  $\overline{q}$  is fixed. The above statements are true for every heat exchanger and for the heat exchange system as a whole.

However, since in the process reality neither of these limiting conditions [for  $\underline{\vec{a}}$  and  $\underline{\vec{q}}$ ] could be implemented, we are allowed to assume that  $\vec{q}$  and  $\underline{\vec{a}}$  are fixed and bounded quantities. In fact, this assumption is equivalent to the acceptance of some constraints which bound the entropy production within an admissible region of the process space. The present study estimates the minimal entropy production for such constrained systems.

Many results stemming from the present theory have previously been known for the special case of two-stream heat exchangers [3,4,5]. Therefore, in what follows, we shall focus on multi-stream heat exchange systems.

The minimal entropy production condition bounds the feasibility of heat exchange systems in the plane with coordinates  $\vec{q}$  and  $\vec{a}$ .

Relation between entropy production and input/output parameters of flowing streams: The total differential of molar entropy expressed in terms of heat capacity and differentials of

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temperature and pressure has the form [2].

$$ds = \frac{C_p}{T} dT - \left(\frac{\partial u}{\partial T}\right)_p dp \tag{1}$$

where T - absolute temperature,  $c_p$  - molar heat capacity at a constant pressure p and v- molar volume. Integrating this equation with respect to temperature and pressure between initial and final values of T and p we shall get the entropy production per one mole. For a known expenditure of a molar flow, a partial entropy production can be attributed to this flow value. By summing these partial values with respect to flows we can get the entropy production in whole system.

For fluids of a constant heat capacity and under constant pressure

$$s - s_o = c_p \ln\left(\frac{T_{\#}}{T_0}\right), \qquad (2)$$

the entropy production due to the change of a state of the  $i^{th}$  flow is equal to the product of its water equivalent and the logarithm of temperature ratio involving input and output temperatures

$$\sigma_i = W_i \ln\left(\frac{T_{\#i}}{T_{0i}}\right), \ i=1, 2, \dots,$$
(3)

Where  $W_i = g_i C_i$  - product of flow rate  $g_i$  and specific heat  $C_i$  (water equivalent). Total entropy production is the sum of entropy productions attributed to individual flows.

## **Double-Flow Heat Exchange**

Let us consider a double-flow heat exchanger, Figure 1, characterized by heat load q and water equivalents  $W_1$  and  $W_2$ . We shall write down present an equation linking the entropy production with system's input temperatures,  $(T_{01}, T_{02})$ , and output temperatures  $(T_{r1}, T_{r2})$ 

$$\sigma = \sigma_{1} + \sigma_{2} = W_{1} \ln \left( \frac{T_{01} - q / W_{1}}{T_{01}} \right) + W_{2} \ln \left( \frac{T_{\#2}}{T_{\#2} - q / W_{2}} \right).$$
(4)

Now we assume that the values of hot flows and heat load are fixed, so that 1 is also fixed. From Eq. (4) a relation follows which links the output temperature of cold flow with the entropy production  $\acute{0}$ 

$$T_{\#2} = \frac{q}{W_2 \left(1 - \exp\left[-\frac{\sigma - \sigma_1}{W_2}\right]\right)}.$$
 (5)

The output temperature of the cold flow increases monotonously with. If is fixed, or q increase, to increase the output temperature of cold flow. The same results hold for multi- flow heat exchangers.



Figure 2: Attainability boundary for the double-flow heat exchanger.

For a double- flow heat exchange with the given heat load these holds the following Proposition is valid

**Proposition 1:** The entropy production of the system with the hot flow

Input temperature  $\boldsymbol{T}_{_{\boldsymbol{0}\boldsymbol{1}}}$  and heat capacity rate  $\boldsymbol{W}_{_{\boldsymbol{1}}}$  cannot be less than

$$\sigma^* = \alpha \frac{\left(1 - m\right)^2}{m},\tag{6}$$

(Figure 2), where

$$m = 1 - \frac{W_1}{\alpha} \ln \frac{T_{01}}{T_{01} - q / W_1}.$$
 (7)

In the case when the cold flow's input temperature  $T_{02}$  and the heat capacity rate  $W_2$  are fixed, the minimal entropy production is

$$\sigma^{*} = \alpha \frac{(n-1)^{2}}{n},$$
(8)  
where  
 $n = 1 + \frac{W_{2}}{\alpha} \ln \frac{T_{02} + q/W_{2}}{T_{02}}.$ 
(9)

Figure 2 shows the minimum entropy production as a function of heat ex-changer heat load  $\mathbf{q}$  [W] and integral heat transfer coefficient  $\mathbf{\dot{a}}$  [W/K]. Values of the temperature of hot flow and its water equivalent are fixed. These estimations are valid for the linear heat transport law

$$q = \alpha (T_1 - T_2)$$

and for every section of the heat exchanger. The minimal entropy production, Eqs (6) and (8), can be achieved in a countercurrent tube heat exchanger with a constant lengthwise exchange coefficient, provided that the flow temperature ratio changes inversely to the heat capacity ratio. In this case, the temperature ratio for every section of the exchanger is

$$\frac{T_1(l)}{T_2(l)} = n = \frac{1}{m}.$$
(10)

The proof of Proposition 1 as a consequence of minimal dissipation conditions was given in [6]. The result was specialized for the linear heat transport law [4]. This result will now be used for the multi- flow heat exchanger.

Let us remark that the condition of constant intensity of sources and constant temperatures of working medium in the heat engine constitutes an optimal condition for the heat engine with finite capacity sources [7].

The ratio of the minimal entropy production and the real entropy production defines the thermodynamic efficiency of a heat exchanger.

As shown in [3], for an arbitrary heat transport law  $q(T_1, T_2)$ , the entropy production is minimal if at each point of hot and cold flows' contact the following conditions hold for  $q^2(T_1, T_2)$ 

$$q^{2}(T_{1},T_{2}) = \lambda \frac{\partial_{q}}{\partial T_{2}} T_{2}^{2}, \qquad (11)$$

Where  $\lambda$  is a constant, which depends on a contact surface and the heat load.

# **Multi Flow Heat Exchange**

A calculation of the complex heat exchange system with several hot and cold flows implies the need to determine flow contact temperatures, the distribution of heat exchange surfaces and heat loads. To solve this problem heuristic algorithms are usually used or the problem is solved only for the given system's structure, [8,13].

In the paper [13] the heat exchange system with three flows and a given structure is considered. It is shown that the entropy production within a countercurrent flow is lower than in a concurrent flow.

Below it is proved that for the system with an arbitrary structure the entropy production is bounded from below and gets attains conditions to reach this bound (Figure 3). The entropy production in the multi- flow heat exchange system with a linear heat transport law is equal to a sum of entropy components:

$$\overline{\sigma} = \sum_{ij} \sigma_{ij} = \sum_{ij} \frac{q_{ij}^2}{\alpha_{ij} \overline{T}_{ij} \overline{T}_{ij}}.$$
(12)

Here the summation is over heat exchanges, each characterized by the index i for hot flows, and the index j for cold flows.

Let us obtain the lower estimate of the entropy production in the multi- flow heat exchange system and corresponding to this low estimate the temperature distributions, as well as distributions of heat exchange coefficients and heat loads in heat exchangers. This estimation permits to find a thermodynamic efficiency

$$\rho = \frac{\sigma}{\sigma} \le 1$$

for a real working system. When designing this system, one can use our calculation to obtain the system's efficiency sufficiently close to that following from our calculations which utilize distributions of temperatures and heat-exchange contact area.

The case when temperatures of hot flows are fixed: Let us consider the input temperature of heating flow  $T_{0i+}$  and the heat capacity rate W  $(T_{0i+}) = W_{i+}$  as fixed quantities. We will denote the heat load for a flow with temperature  $T_{0i+}$  as  $q_i$ , and the corresponding heat exchange coefficient as  $\alpha_i$ .

The distribution of heat exchange surfaces is equal to the distribution of the heat exchange coefficients, therefore the following coefficient

$$\alpha = \sum_{i} \alpha_{i}, \tag{13}$$

will be assumed as a constant quantity. We shall also use the sum of heat loads

$$\overline{q} = \sum_{i} q_i \tag{14}$$

When  $T_{_{i+}}$ ,  $W_{_{i+}}$  and q are fixed, then the average enthalpy of hot streams in the outlet stream of the system is fixed as well.

Outlet temperatures of hot flows depend on inlet temperatures and the heat load as follows

$$T_{\#_{i+}} = T_{0i+} - q_i / W_{i+} \quad i = 1, \dots, n.$$
(15)

The problem of entropy production minimization has the form

$$\overline{\sigma} = \sum_{i} \sigma_{i} \rightarrow \min_{u(T_{i+}, T_{oi^{+}}), \alpha_{i}, q_{i}}$$
(16)

where u ( $T_{i+}$ ;  $T_{0i+}$ ) the contact temperature of the cold flow which exchanges the energy with the hot flow of the input temperature  $T_{0i+}$  and the current temperature  $T_{i+}$ .

**Derivation of computational relations:** We shall solve the problem (13)-(16) in two stages. At the first stage, we assume that  $q_i$  and  $\alpha_i$  are fixed for all i. Keeping these conditions in mind, we de ne the correlations of current temperatures for the heated flows  $u_i$ , and the heating flows  $T_{i+}$  corresponding to the minimal entropy production  $\delta_i$  [denoted in Eq. (8) by ] for the heating flow with initial temperature  $T_{0+}$ .

At the second stage, we de ne the contact surface distribution i and the heat load distribution qi, both capable to minimize subject to the constraints (13) and (14).

The first problem is solved in section 2, Eqs (8) and (9). For every inlet temperature of a hot stream

$$m_{i} = \frac{u(T_{i-,}T_{oi-})}{T_{i+}} = 1 - \frac{W_{i}}{\alpha_{i}} \ln \frac{T_{oi+}}{T_{oi+} - \frac{q_{i}}{W_{i+}}}, \quad i=1,2,...,n \quad (17)$$

$$\sigma_{i}^{*} = \alpha_{i} \frac{(1-m_{i})^{2}}{m_{i}}, \quad i=1,2,...,n \quad (18)$$

The second stage results in  $\alpha$  and q distribution problem subject to the constraints (13), (14) and the condition

$$\overline{\sigma} = \sum_{i} \sigma_{i}^{*} \left[ T_{oi+,} \alpha_{i} W_{i+,} q_{i} \right] \rightarrow \min_{\alpha \ge 0, q \ge 0} \quad (19)$$

Here the Lagrange function takes the following form

$$L = \sum_{i} \sigma_{i}^{*}(T_{oi+,}\alpha_{i},W_{i+,}q_{i}) - \lambda_{1}\sum_{i} \alpha_{i} - \lambda_{2}\sum_{i} q_{i}.$$

where  $\lambda_1$  and  $\lambda_2$  are some constants independent of i

The stationary conditions for L lead to equations

$$\frac{\partial \sigma_i}{\partial \alpha_i} = \lambda_1, \frac{\partial \sigma_i}{\partial q_i} = \lambda_2 \forall i \tag{20}$$

In order to calculate derivatives in (20), let us extract the following derivatives

$$\frac{\partial m_i}{\partial \alpha_i} = \frac{W_i}{\alpha_i^2} \ln \frac{T_{oi+}}{T_{\#i+}} = \frac{1 - m_i}{\alpha_i}$$

$$\frac{cm_i}{\partial q_i} = -\frac{1}{\alpha_i T_{\#i+}},$$

$$\frac{\partial \sigma_i^*}{\partial m_i} = \alpha_i \frac{m_i^2 - 1}{m_i^2}.$$
Hence Eq. (20) takes the following form
$$\frac{\partial \sigma_i^*}{\partial \alpha_i} = -\left(\frac{1 - m_i}{m_i}\right)^2 = \lambda_1,$$

$$\frac{\partial \sigma_i^*}{\partial q_i} = -\frac{m_i^2 - 1}{m_i^2 T_{\#i+}} = \lambda_2, i = 1, 2, \dots, n$$

$$T_{\#i+} = \frac{1 - m_i^2}{m_i^2 \lambda_2}.$$
(21)
(22)

From (21) it follows that for the optimal heat exchange organization, m is independent of  $T_{0i+}$  and the output temperatures for all flows should be equal to each other:

 $T_{#i+} = T_{#i+}^*$ :

 $T^*_{#+}$  is unambiguously determined by Eq. (14) since

$$\overline{q} = \sum_{i} W_{i}(T_{oi+} - T_{\#+}^{*}).$$
(24)
After defining
$$\overline{W_{+}} = \sum W_{i+}.$$
(25)

$$\overline{T_{o+W}} = \sum_{i} T_{oi+} W_{i+,}$$
(26)

We obtain

$$T_{\#+}^{*} = \frac{\overline{T_{o+}W} - \overline{q}}{\overline{W_{+}}}.$$
(27)

In order to express m by initial data rewrite Eq. (17) as follows

$$\alpha_i = \frac{W_i + (\ln T_{oi+} - \ln T_{\#+}^*)}{1 - m}.$$
(28)

As  $a_i$  is non-negative then  $T_{0i+} \ge T^*_{\#+}$ .

Therefore in a heat exchange system only hot flows with a temperature higher than  $T^*_{_{\#+}}$  should be used. If  $T_{_{0i+}} < T^*_{_{\#+}}$  then all contact surface areas for this flow should be zero.

This result corresponds to the fact that, in optimal cycles of the heat engine with several sources, it is not advantageous to contact with hot sources having temperatures lower then  $T_{+min}$  and cold sources with temperature greater then T max (see [14]).

After summing the left and the right hand sides of (28), where the integral heat exchange coefficient is fixed, we find m in the form

$$m = 1 - \frac{1}{\alpha} \sum_{i} W_{i+} (\ln T_{oi+} - \ln T_{\#+}^*).$$
(29)  
Thus the optimal distribution of heat exchange coefficient is  
$$\alpha_i = \alpha \frac{W_{i+} (\ln T_{oi+} - \ln T_{\#+}^*)}{\sum_{i} (\ln T_{oi+} - \ln T_{\#+}^*)},$$
(30)

The heat load distribution is

$$q_i = W_{i+}(T_{oi+} - T_{\#+}^*), \tag{31}$$

and the least possible entropy production is

$$\sigma^* = \overline{\alpha} \frac{(1-m)^2}{m}.$$
 (32)

After comparing Eq. (32) with the entropy production  $\overline{6}$  of a real



heat exchange system characterized by total heat exchange coefficient  $\overline{a}$ , hot flow inlet temperatures  $T_{_{0i+}}$ , heat capacity rates  $W_{_{i+}}$  and the outlet enthalpies of heating flows  $W_{_{i+}}$ ,  $T_{_{\#i+}}$  we can estimate the system's efficiency as  $\rho = \frac{\sigma}{\sigma}$ .

In order to make the system characteristics closer to ideal, we should distribute the heating flows and the heat exchange surfaces according to Eq's (31) and (30) and choose contact temperatures according to Eq. (29) i.e. the condition of temperature constancy in terms of m. This can be done by reducing the heat exchange surface in heat exchangers with characterized by temperature ratio for hot and cold flows higher than the system's average value. For heat exchangers with the temperature ratio lower than the average value, the heat exchange surface should be enlarged. Similarly, the heat intake should be increased for heating flows with output temperatures higher than the average outlet temperature of heating flows.

The final formulas describing the optimal choice for output temperatures of heated flow, the heat load, the heat exchange coefficient, the temperature ratio for contact flows and the least possible dissipation take the following form:

$$T_{\#+}^{*} = \frac{\sum_{i=1}^{k} T_{io+} W_{i+} - \overline{q}}{\sum_{i=1}^{k} W_{i+}},$$

$$q_{o}^{*} = W_{i+} (T_{io+} - T_{\#+}^{*}),$$

$$\alpha_{o}^{*} = \frac{\overline{\alpha} W_{i+} (\ln T_{io+} - \ln T_{\#+}^{*})}{\sum_{i=1}^{k} W_{i+} (\ln T_{io+} - \ln T_{\#+}^{*})},$$

$$m = 1 - \frac{1}{\overline{\alpha}} \sum_{i} W_{i+} (\ln T_{oi+} - \ln T_{\#+}^{*}),$$

$$\overline{\sigma}^{*} = \overline{\alpha} \frac{(1-m)^{2}}{m},$$

$$\alpha_{o}^{*} = q_{o}^{*} = W_{i} = 0, T_{io+} \leq T_{\#+}^{*},$$
(33)

where k is the number of hot flows, which it is advisable to use  $(T_{i0+} > T_{*+})$ .

Therefore the proposition 2a has been proved.

**Proposition 2a:** For heat exchange systems with given number of hot flows, inlet temperatures, heat capacity rates, total heat load and heat exchange surface in a linear heat transport law, the entropy production satisfies inequality  $\sigma \ge \sigma^*$ .

If conditions (33) hold, then inequality turns into equality.

**Temperatures of cold flows are fixed:** To estimate  $\sigma^*$  we assume

that characteristics of hot flows are given and the task is to select cold flows parameters in such a way that the entropy production is minimal. But it may also happen that characteristics of cold flows are given.

We shall denote variables of cold flows (temperatures, heat capacity, rates, etc.) With the index "-". Using previous arguments, we can find the value of minimal entropy production for a system with fixed temperatures of cold flows,  $T_{1.} \ge T_{1.} \ge T_{2.}$  and heat capacity rates,  $W_{i.}$ . Below we present the computation relations derived for this case.

$$T_{\#+}^{*} = \frac{\sum_{i=1}^{k} T_{i-}W_{i-} + \overline{q}}{\sum_{i=1}^{k} W_{i-}},$$

$$q_{i-}^{*} = W_{i-}(T_{\#-}^{*} - T_{i-}),$$

$$\alpha_{i-}^{*} = \frac{\overline{\alpha}W_{i-}(\ln T_{\#-}^{*} - \ln T_{i-})}{\sum_{i=1}^{k} W_{i-}(\ln T_{\#-}^{*} - \ln T_{i-})},$$

$$\frac{1}{m} = 1 + \frac{1}{\overline{\alpha}} \sum_{i=1}^{k} W_{i-}(\ln T_{\#-}^{*} - \ln T_{i-}),$$

$$\overline{\sigma}^{*} = \overline{\alpha} \left(\frac{1}{m} + m - 2\right),$$

$$\alpha_{-}^{*}(T_{i-}) = q^{*}(T_{i-}) = W_{i-} = 0, T_{i-} \ge T_{\#+}^{*}.$$
(34)

The last requirement states that in the system with the minimal entropy production all input temperatures of cold flows should neither exceed nor be equal to  $T^*_{\mu}$ .

**Proposition 2b:** For heat exchange systems with given number of cold flows, inlet temperatures, heat capacity rates, total heat load and heat exchange surface, the entropy production satisfies inequality  $\sigma \ge \sigma_{-}^{*}$  which means that

$$\sigma \ge \sigma^*$$
. (35)

This inequality is a thermodynamic feasibility condition satisfied in multi flow heat exchange systems. Heat exchangers for which this condition is violated cannot be implemented.

In a real system parameters of cold and hot flows are usually given. To evaluate the thermodynamic efficiency of such system we should take use the maximal value of  $\sigma^*$  or  $\sigma^*_-$ , to get more detailed estimate of efficiency.

#### Example

Figure 4 shows the heat exchange system with three hot and cold flows. Hot flows index is +, whereas cold flows index is - . Temperatures, in Kelvins, are shown on the figure next to the arrows. Heat exchange coefficients [kJ/sK] are given in circles, and flows of water equivalents for each inlet flow are  $W_{i+}$  and  $W_{i-}$ .

Assume that the effective temperature is equal to the mean temperature of the input and the output temperature in each heat exchanger. For these conditions we get heat loads  $q_{ij}$  for each heat exchanger (Table1). In accordance with Eq. (4), the entropy production in this system, calculated as the sum of entropy production

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Figure 4: The heat exchange system

Table 1: Heat loads q

j\i	1	2	3
1	885	416	271.5
2	591.9	375.3	452
3	1233.5	983.7	549.2

increases for each flow, is described by the equation

The entropy production in this system (4) calculated as sum of entropy production increases for each flow.

$$\sigma = \sum_{i=1}^{3} W_{i+} \ln \frac{T_{i\#+}}{T_{io+}} + \sum_{j=1}^{3} W_{i-} \ln \frac{T_{j\#-}}{T_{j0-}}.$$
(36)

The numerical value of the entropy production is equal to = 5; 574 [kJ/sK]. We use the set of formulas in Eq. (33) to perform calculations of the optimal thermodynamic system with total heat load q=5851 [kJ/s] and heat exchange coefficients [in conductance units] $\overline{a}$  =48 [kJ/sK]. We will get the optimal out-put temperature for hot flows T<sup>\*</sup><sub>\*+</sub> = 453.4 K. Comparing this temperature with inlet temperatures for hot flows, we conclude that the third flow with the

Temperature 450 K should be excluded from the system.

Recalculation of  $T_{g_+}^*$  for two hot flows for the same values of q and  $\tilde{a}$  gives  $T_{g_+}^* = 457.2$ . Optimal values of heat loads for first and second flows are respectively equal to  $q(T_{01+}) = 3712[kJ/s]$  and  $q(T_{02+}) = 2140 [kJ/s]$ ; the optimal contact surface distribution for these two flows, Eq. (33), corresponds with  $\tilde{a}$   $(T_{01+}) = 29.9 [kJ/sK]$ ,  $\tilde{a}$   $(T_{02+}) = 18,1 [kJ/sK]$ . The ratios of effective temperatures should be equal for cold and hot flows, and are equal to m=0.752. The minimal entropy production in this system is equal to  $\sigma^* = 3.93 [kJ/sK]$ . For this system the thermodynamic efficiency is equal to

$$\rho = \frac{\sigma}{\sigma} = 0,705.$$

Combining results for real and optimal systems we can formulate the following recommendations:

1. The flow with the input temperature 450K should be removed from the system. Heat exchange surfaces for other flows grow for the first flow from 19 to 30, for the second one from 14 to 18 [kJ/sK].

2. Heat exchange surfaces for each flow should be distributed in

such a way that the ratio of effective temperatures is close to 0.75. Note that in the real system this value changed from 0.63 to 0.88.

3. The output temperatures of hot flows should be close to 457,2.

#### Conclusion

In this paper we de ne the thermodynamically optimal organization of the heat exchange in order to achieve the least possible entropy production for a system with the fixed heat load and the fixed total heat transfer coefficient. We also define the appropriate heat load distribution and the heat exchange coefficient distribution for the input flows. Our study permits to estimate the thermodynamic efficiency for a random heat exchange system and to refine it, as well as to analyze the system's dependence on such factors as the temperature variations of flows or changes in the heat exchange surfaces.

Optimal conditions for a heat exchanger cannot be implemented for some systems for any distribution of heat exchange surfaces. It means that the structure parameters of these systems should be changed.

Analogous results can be obtained for other heat transport laws. In this case the condition of the least possible dissipation should be used for a given but different kinetics.

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